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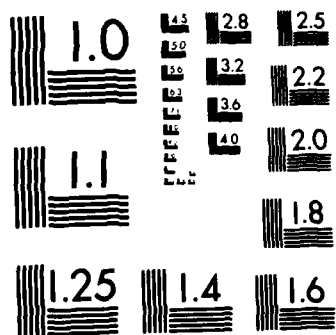
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The main thrust of <sup>THIS</sup>our research program has been the development and applications of asymptotic and perturbation methods for analyzing: the stability and dynamics of elastic structures, fluid flow, and other nonlinear problems; and for problems of scattering of acoustic, electro-magnetic and other waves. The work is summarized in the following papers which have been published, accepted for publication, submitted for publication, or are in preparation for publication. ★

1. Erneux, T., and Reiss, E. L., Brussellator Isolates, SIAM J. Appl. Math., 43 (1983), pp. 1240-1246.

The Brussellator is a simple chemical model describing pattern formation by bifurcation of solutions. For a one-dimensional system, the bifurcation parameter is related to the ratio of the square of the size of the system to a diffusion coefficient. It has been observed from numerical computations, that there are closed branches of steady state solutions, which are called isolas, that connect neighboring bifurcation points. In addition, these isolas depend on a parameter  $B$ . As  $B$  approaches a critical value  $B^0$  the neighboring bifurcation points coalesce, so that the isola shrinks to a point. We employ a perturbation method to obtain asymptotic expansions of the isolas for  $B$  near  $B^0$ . Implications of the results for pattern formation are discussed.

2. Kriegsmann, G. A., and Reiss, E. L., Acoustic Propagation in Wall Shear Flows and the Formation of Caustics, J. Acoust. Soc. Amer., 74 (1983), pp. 1869-1879.

The propagation of acoustic waves from a high frequency line source in a two dimensional parallel shear flow adjacent to a rigid wall is analyzed by a ray method. The leading term in the resulting expansion is equivalent to the geometrical acoustics theory of classical wave propagation. It is shown that energy from the source is radiated either directly to the far field, or by first reflecting from the wall. In addition, energy is trapped in a channel adjacent to the wall and downstream from the source. The rays in this channel form an infinite sequence of caustics progressing downstream. Since the geometrical acoustics approximation is invalid on and near caustics, a boundary layer method is employed to determine the acoustics field near the caustics. It is shown that the amplitude of the field on and near the caustics is  $k^{2/3}$  larger than the geometrical acoustics field for large  $k$ . Here  $K$  is a dimensionless wave number of the source. Moreover, the vorticity of the acoustics field in the caustic regions is  $k^{7/6}$  larger than the geometrical acoustics field. The possible significance of these results for vehicle self-noise and the formation of turbulent spots in the sub-layer of a turbulent boundary layer is discussed.

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MATTHEW J. KENNER

Chief, Technical Information Division

3. Erneux, T., and Reiss, E. L., Splitting of Steady Multiple Eigenvalues May Lead to Periodic Bifurcation, SIAM J. Appl. Math., 43 (1983), pp. 613-624.

A general bifurcation problem is considered that depends on two parameters in addition to the bifurcation parameter  $\lambda$ . It is assumed that all primary bifurcation states correspond to steady solutions and that they branch supercritically. Then it is shown that for a range of system parameters, and near a triple primary bifurcation point the following cascade of bifurcations from the minimum primary bifurcation state is possible. As  $\lambda$  increases there is secondary and then tertiary bifurcation to steady states, and finally Hopf bifurcation at a quaternary bifurcation point. Related transitions have been observed experimentally in thermal convection and other hydrodynamic stability problems. In addition, we show that Hopf bifurcation near a double primary bifurcation point is not possible when both primary states near the double point bifurcate supercritically. However, it is possible near such a double bifurcation point if imperfections are included in the formulation, as we demonstrate.

4. Grotberg, J. B., and Reiss, E. L., Subsonic Flapping Flutter, J. Sound and Vibration, 92 (1984), pp. 349-361.

A mathematical model of two-dimensional flow through a flexible channel is analyzed for its stability characteristics. Linear theory shows that fluid viscosity, modelled by a Darcy friction factor, induces flutter instability when the dimensionless fluid speed,  $S$ , attains a critical flutter speed,  $S_0$ . This is in qualitative agreement with experimental results, and it is at variance with previous analytical studies where fluid viscosity was neglected and divergence instability was predicted. The critical flutter speed and the associated critical flutter frequency depend on three other dimensionless parameters: the ratio of fluid to wall damping; the ratio of wall to fluid mass; and the ratio of wall bending resistance to elastance. Non-linear theory predicts stable, finite amplitude flutter for  $S > S_0$ , which increases in frequency and amplitude as  $S$  increases. Both symmetric and antisymmetric modes of deformation are discussed.

5. Erneux, T. and Reiss, E. L., Singular Secondary Bifurcation, SIAM J. Appl. Math., 44 (1984), in press.

A bifurcation problem is analyzed for a Brusselator boundary value problem, which is a typical reaction-diffusion system. The bifurcation parameter  $\lambda$  is proportional to the length of the system. We employ previously developed perturbation methods to analyze the secondary bifurcation of steady solutions that arise from the splitting of multiple primary bifurcation points. The resulting bifurcation equations are nonlinear algebraic equations to determine the amplitudes of the solutions. The coefficients in these equations depend upon the system parameters in the Brusselator problem, such as the two prescribed constant reactants A and B. For critical values of these parameters the solutions of the algebraic system are singular, and hence the perturbation method is invalid for parameter values at and near these critical values. A new perturbation method is employed yielding new branches of steady solutions and possible tertiary bifurcations to time-periodic solutions. Thus the analysis of singularities in the bifurcation equation reveals new mechanisms for the occurrence of steady and time-periodic solutions. Some implications of the results for chemical morphogenesis are discussed.

6. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Acoustic Scattering by a Baffled Membrane, J. Acoust. Soc. of Amer., 75 (1984), pp. 685-694.

A flexible membrane is set in an infinite plane baffle. The plane separates an acoustic fluid from a vacuum. A time harmonic wave is incident from the fluid on the membrane. When the frequency of the incident wave is not close to an in-vacuo resonant frequency of the membrane, the reaction of the fluid on the membrane is small. However near a resonant frequency the fluid-membrane coupling is significant. We use the method of matched asymptotic expansions to obtain an asymptotic expansion of the scattered field. It is uniformly valid in the incident frequency. The expansion parameter  $\epsilon \ll 1$  is the ratio of the fluid and membrane densities. The outer expansion, valid away from resonance, is  $O(\epsilon)$ . The inner expansion valid near resonance is of order unity. The fluid loading is shown to have the effect of decreasing the resonant frequencies from those of the in-vacuo membrane. Simple and double resonant frequencies are analyzed. However, the method is applied to normal incidence of a plane wave on a circular membrane.

7. Erneux, T., Matkowsky, B. J. and Reiss, E. L., Singular Bifurcation in Reaction-Diffusion Systems, in "Modelling of Patterns in Space and Time," Ed. by J. Murray and W. Jager, Springer, 1984, pp.67-72.

Bifurcation Theory is a study of the branching of solutions of equations as a parameter  $\lambda$ , called the bifurcation parameter, is varied. The branching, or bifurcation points, are singular points of the solutions. In Bifurcation Theory solutions are analyzed near bifurcation points. In this theory there is usually a distinguished solution which is called the basic state. Typically, it exists for all values of  $\lambda$  and it is usually determined by the physics, or biology that the bifurcation problem models. The bifurcation points of the basic state are the primary bifurcation points and the solutions branching from them, other than the basic state, are the primary bifurcation states. These states are determined analytically by asymptotic methods, such as the Poincare-Linstedt or other equivalent methods. The results of these asymptotic analyses yield a system of equations, simpler than the original bifurcation problem, to determine the amplitudes of the primary bifurcation states. They are called the primary amplitude equations. The primary bifurcation states are usually more complex spatially and/or temporally than the basic state, thus initiating a pattern formation. Similarly, solutions of the bifurcation problem, other than the basic state, that branch from the primary bifurcation states are secondary bifurcation states and the corresponding branching points are secondary bifurcation points. The secondary bifurcation states are frequently more complex than the primary bifurcation states, thus continuing the pattern formation. More generally, there may be a sequence of bifurcations whose corresponding solutions are increasingly more complex spatially and temporally. We have referred to this elsewhere as cascading bifurcation. The idea of cascading bifurcation has been proposed as a possible explanation of the transition from laminar

to turbulent flows in certain hydrodynamic stability problems as a flow parameter  $\lambda$ , such as the Reynolds number, increases. Similarly, cascading bifurcations have been offered as a theoretical explanation of biological and biochemical morphogenesis. In many problems the primary bifurcation points, which we denote by  $\lambda_1(p), \lambda_2(p), \dots$ , depend on a vector  $p$  of auxiliary parameters that occur in the original bifurcation (reaction-diffusion) problem. It is customary to analyze secondary and cascading bifurcation by considering the additional singular situation when two or more primary bifurcation points coincide as the vector  $p \rightarrow p_0$  to produce a multiple primary bifurcation point. Then asymptotic and other methods are employed to determine the cascading bifurcation of solutions, where the small parameter in the asymptotic analysis is a monotone function of  $|p - p_0|$ . The results of the analysis yield amplitude equations for the secondary and higher order states. It has recently been recognized in certain reaction-diffusion problems that the primary or secondary amplitude equations are singular i.e. they are not uniquely solvable as  $p$  approaches special values  $p_1$ . We refer to this situation as singular bifurcation (primary or secondary) or more generally singular cascading bifurcation. Then the asymptotic expansions that have been obtained are not uniformly valid as  $p \rightarrow p_1$ .

8. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Scattering by Penetrable Acoustic Targets, Wave Motion, 6 (1984), pp. 501-516.

An acoustic target of constant density  $\rho_t$  and variable index of refraction is imbedded in a surrounding acoustic fluid of constant density  $\rho_a$ . A time harmonic wave propagating in the surrounding fluid is incident on the target. We consider two limiting cases of the target where the parameter  $\epsilon \equiv \rho_a/\rho_t \rightarrow 0$  (the nearly rigid target) or  $\epsilon \rightarrow \infty$  (the nearly soft target). When the frequency of the incident wave is bounded away from the "in-vacuo" resonant frequencies of the target, the resulting scattered field is essentially the field scattered by the rigid target for  $\epsilon = 0$  or the soft target if  $\epsilon \rightarrow \infty$ . However, when the frequency of the incident wave is near a resonant frequency, the target oscillates and its interaction with the surrounding fluid produces peaks in the scattered field amplitude. In this paper we obtain asymptotic expansions of the solutions of the scattering problems for the nearly rigid and the nearly soft targets as  $\epsilon \rightarrow 0$  or  $\epsilon \rightarrow \infty$ , respectively, that are uniformly valid in the incident frequency. The method of matched asymptotic expansions is used in the analysis. The outer and inner expansions correspond to the incident frequencies being far or near to the resonant frequencies, respectively. We have applied the method only to simple resonant frequencies, but it can be extended to multiple resonant frequencies. The method is applied to the incidence of a plane wave on a nearly rigid sphere of constant index of refraction. The far field expressions for the scattered fields, including the total scattering cross-sections, that are obtained from the asymptotic method and from the partial wave expansion of the solution are in close agreement for sufficiently small values of  $\epsilon$ .



9. Bayliss, A., Kriegsmann, G. A. and Morawetz, C. S., The Nonlinear Interaction of a Laser Beam with a Plasma Pellet, Comm. on Pure and Appl. Math. 36 (1983), pp. 399-414.  
  
A model is developed which simulates the interaction of a laser beam with a plasma. The model allows for a nonlinear dependence of the density on the amplitude of the electric field. Numerical results show that for strong nonlinearities time-harmonic fields cannot be obtained. Instead pulses of electromagnetic energy (solitary waves) are formed inside the plasma and carry energy away from the plasma.
10. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., An Asymptotic Analysis of Acoustic Scattering by Nearly Rigid or Soft Objects, in "Wave Motion: Modern Theory and Application," Edited by C. Rogers and T. Moodie, Elsevier, 1984.
11. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Can Acoustic Focusing Generate Turbulent Spots?, in "Wave Motion" Modern Theory and Application, edited by C. Rogers and T. Moodie, Elsevier, 1984.
12. Erneux, T., Mandel, P. and Magnan, J. F., Quasi-Periodicity in Lasers with Saturable Absorbers, Phys. Rev. (A), 29 (1984), pp. 2690-2694.

In this paper, we consider the mean-field equations for the laser with a saturable absorber (LSA) and concentrate on the low-intensity solutions. We show that the LSA equations may admit two successive bifurcations: the first bifurcation corresponds to the transition from the zero-intensity state to time-periodic intensities and is a Hopf bifurcation. The second bifurcation corresponds to the transition from these time-periodic intensities to quasi-periodic intensities which are characterized by two incommensurable frequencies. In order to describe these transitions, we investigate a particular limit of the parameters and propose a new perturbation method for solving the LSA equations. We give analytical conditions for the existence of both the primary and secondary bifurcations.

13. Mandel, P. and Erneux, T., Stationary, Harmonic and Pulsed Operations of an Optically Bistable Laser with Saturable Absorber, Part I, Phys. Rev. A 30 (1984), pp. 1893-1901.

We study the semiclassical equations for a laser with a saturable absorber in the mean-field limit, assuming homogeneously broadened two-level atoms, for a set of parameters where the system displays optical bistability and time-periodic solutions. In the first part the bifurcation diagram for stationary and periodic solutions is obtained by numerical integration. Two different classes of stable periodic solutions arise: small-amplitude solutions and passive Q switching. We observe hysteresis domains involving up to three solutions (stationary and/or periodic). We also discuss the validity of some standard approximations and show that even in the absence of detuning the phases play an important role. We also discuss the influence of the initial conditions whose symmetry properties induce important modifications of the bifurcation diagram. In the second part we introduce an alternative adiabatic elimination scheme which allows us to construct the small-amplitude periodic solutions over nearly their whole range of existence. We then study these solutions near the Hopf bifurcation from which they emerge and derive analytic conditions for their stability. When they are stable, we also give the conditions under which a secondary Hopf bifurcation will occur, leading to quasiperiodic solutions.

14. Erneux, T. and Mandel, P., Stationary, Harmonic and Pulsed Operations of an Optically Bistable Laser with Saturable Absorber, II, Phys. Rev. A 30 (1984), pp. 1902-1909.

In the preceding paper we have analyzed the bifurcation diagram of the steady and time-periodic solutions of the lasers with saturable absorbers (LSA) equations. However, a study of the experimental results presented in the literature indicates that, in general, the control parameter is a slowly varying function of time. In this second paper we analyze the influence of this time dependence on the bifurcation diagram of the LSA. We show that the stability changes of the slowly varying steady-state solutions do not correspond to their bifurcation or limit points in the case where all parameters are constant. In particular, we show that the zero-intensity state can be stabilized during a certain interval of time and that this stabilization can be controlled by the initial value of the time-dependent bifurcation parameter.

15. Magnan, J. and Reiss, E. L., Double-Diffusive Convection and  $\lambda$ -Bifurcation, Physical Review A, in press.

We consider convection in a rectangular box where two "substances" such as temperature and a solute are diffusing. The solutions of the Boussinesq theory depend on the thermal and solute Rayleigh numbers  $R_T$  and  $R_S$  in addition to other geometrical and other fluid parameters. The conduction state is unstable with respect to steady (periodic) convection states if  $R_S$  is sufficiently small (large). The boundary between steady and periodic convection occurs at a critical value  $R_S = \bar{R}_S$ . The linearized theory at  $R_S = \bar{R}_S$  is characterized by the frequency  $\omega = 0$  appearing as a root of algebraic multiplicity two and geometrical multiplicity one. Asymptotic approximations of the solutions are obtained for  $R_S$  near  $\bar{R}_S$  by the Poincare-Linstedt method. It is found that a periodic (steady) solution bifurcates supercritically (subcritically) from the conduction state at  $R_T = R_T^P(R_S^S)$ , where  $R_T^P < R_T^S$ . The periodic branch joins the steady branch with an "infinite period bifurcation" at  $R_T = R_b$ , where  $R_T^P < R_b < R_T^S$ . The shape of the resulting bifurcation diagram suggests the term, bifurcation. The infinite periodic bifurcation corresponds to a heteroclinic orbit in the appropriate amplitude phase plane. The periodic (steady) convection states are stable (unstable), as we demonstrate by solving the initial value problem employing the multi-scale method.

16. Grotberg, J. B. and Reiss, E. L., Secondary Bifurcation of Quasiperiodic Solutions Can Lead to Period Multiplication, SIAM J. Appl. Math., in press.

It is shown using perturbation and asymptotic methods that secondary bifurcation of quasi-periodic solutions from periodic solutions occurs for a model problem. The model is a coupled system of two van der Pol-Duffing oscillators. For special values of the detuning parameters the secondary states are periodic. Then periodic multiplication of solutions can occur at the secondary bifurcation point.

17. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., An Optical Theorem for Acoustic Scattering by Baffled Flexible Surfaces, J. Sound and Vib., in press.

The classical optical theorem for scattering by compact obstacles is a forward scattering theorem. That is, the total cross-section of the obstacle is proportional to the imaginary part of the far field directivity factor evaluated in the forward scattering direction. We derive an analogous theorem for the scattering of acoustic waves by baffled membranes and plates. In this "optical" theorem the directivity factor is evaluated in the direction of the specularly reflected wave, so that it is a reflected scattering theorem.

18. Porter, M. B. and Reiss, E. L., A Numerical Method for Acoustic Normal Modes for Shear Flows, J. Sound and Vib., in press.

The normal modes and their propagation numbers for acoustic propagation in wave guides with flow are the eigenvectors and eigenvalues of a boundary value problem for a non-standard Sturm Liouville problem. It is non-standard because it depends nonlinearly on the eigenvalue parameter. (In the classical problem for ducts with no flow, the problem depends linearly on the eigenvalue problem). In this paper we present a method for the fast numerical solution of this problem. It is a generalization of a method that was developed for the classical problem. A finite difference method is employed that combines well known numerical techniques and a generalization of the Sturm sequence method to solve the resulting algebraic eigenvalue problem. Then a modified Richardson extrapolation method is used that dramatically increases the accuracy of the computed eigenvalues. The method is then applied to two problems. They correspond to acoustic propagation in the ocean in the presence of a current, and to acoustic propagation in shear layers over flat plates.

19. Lange, C. G. and Kriegsmann, G. A., Bifurcation Analysis of Nonlinear Turning Point Problems, SIAM J. Appl. Math., in press.

A bifurcation analysis is carried out on a class of nonlinear two-point boundary value problems for which the associated linearized equations have turning point structure. A perturbation method is used to study the behavior of solutions branching from large eigenvalues. The results are compared with those previously obtained for problems without turning points.

20. Erneux, T. and Mandel, P., Imperfect Bifurcation with a Slowly-Varying Control Parameter, SIAM J. Appl. Math., in press.

We consider a general class of imperfect bifurcation problems described by the following first order nonlinear differential equation:

$$y_t = k^p + \lambda(t)y + \delta$$

where  $k = 1$  or  $-1$  and  $p = 2$  or  $3$  are fixed quantities. The solution depends on the value of the "imperfection" parameter  $\delta (0 < \delta \ll 1)$  and the time-dependent control parameter  $\lambda(t) = \lambda_0 + \epsilon t$  ( $\lambda_0 < 0$  and  $0 < \epsilon \ll 1$ ). If  $\delta = \epsilon = 0$ , this equation admits at  $\lambda = 0$  a bifurcation from the basic state  $y = 0$  to non-zero steady states. In the first part of the paper, we analyze the perturbation of the bifurcation solutions produced both by the small imperfection ( $\delta \neq 0$ ) and the slow variation of ( $\epsilon \neq 0$ ). We show that  $\lambda = 0$  does not correspond to the transition between the two branches of slowly-varying steady states. This transition appears at a larger value of  $\lambda = \lambda_1$ . Provided that  $\delta$  is sufficiently small compared to  $\epsilon$ ,  $\lambda_1$  is an  $O(1)$  quantity which only depends on  $\lambda_0$ , i.e., the initial position of  $\lambda(t)$ . Our analysis is motivated by problems appearing in laser physics. In the second part of the paper, we show how the semi-classical equations for the simple laser and the laser with a saturable absorber can be reduced to this simple first-order nonlinear equation. We then discuss the practical interests of our results.

21. Strumolo, G. and Reiss, E. L., Poiseuille Channel Flow with Driven Walls, J. Fluid Mech., submitted.

The effects of a prescribed wall motion on the nonlinear stability of Poiseuille channel flow are studied by an asymptotic method. The motion represents a traveling wave in the upper wall of the channel. It can be considered either as a disturbance to the flow that results from experimental imperfections, or as an externally imposed motion. The frequency of this disturbance depends on the Reynolds number of the flow. In the classical Poiseuille channel flow problem, the walls are assumed to be rigid. Then a periodic solution bifurcates from the laminar, Poiseuille flow at the critical Reynolds number  $R_c$ . In the resonance case, the wall motion destroys the bifurcation. The transition from the laminar state then occurs by jumping at new critical Reynolds numbers. These Reynolds numbers either exceed, or are below  $R_c$ , depending on the variation of the wall motion frequency with the Reynolds number. Thus the wall motions can stabilize or destabilize the laminar flow, and hence they can be used to control the transition to turbulence. In the non-resonance cases, the bifurcation is preserved and the critical Reynolds number is slightly perturbed.

22. Magnan, J. and Reiss, E. L., Rotating Thermal Convection: Neo-Periodicity and Escape, J. Fluid Mech., submitted.

We consider the secondary and cascading bifurcation of two-dimensional steady and periodic thermal convection states in a rotating box. We employ previously developed asymptotic and perturbation methods that rely on the coalescence of two, steady convection, primary bifurcation points of the conduction state as the Taylor number approaches a critical value. A multi-time analysis is employed to construct asymptotic expansions of the solutions of the initial-boundary value problem for the Boussinesq theory. The small parameter in the expansion is proportional to the deviation of the Taylor number from its critical value. To leading order, the asymptotic expansion of the solution involves the mode amplitudes of the two interacting steady convection states. The asymptotic analysis yields a first order system of two coupled ordinary differential equations for the slow-time evolution of these amplitudes. We investigate the steady states of these amplitude equations and their linearized stability.

These equations suggest that the following sequence of transitions may occur as the Rayleigh number  $R$  is increased, for a fixed value of the Taylor number near the critical value: First, the conduction state loses stability at a primary bifurcation point to steady convection states (rolls) characterized by a single wavenumber. Then, these states lose stability at a secondary bifurcation point to other steady convection states characterized by two different wavenumbers. Finally, these states become unstable at a tertiary Hopf bifurcation point,  $R = R_H$ , to time-periodic convection states, which are also characterized by the same two wavenumbers. The Hopf bifurcation is degenerate since for  $R = R_H$  there is a one-parameter family of periodic solutions, which is bounded in the phase plane by a heteroclinic orbit.

For  $R \neq R_H$  but close to  $R_H$ , the center of the system's phase plane trajectories is transformed into a focus which is unstable (stable) for  $R > R_H$  ( $< R_H$ ). We find that, depending on the length of the observation time and on the initial conditions, the system may appear to be either in a single steady state, or jumping between multiple steady states, or in a periodic state, or in a transient state. If the observation time is sufficiently long the trajectories ultimately spiral into the focus for  $R < R_H$  and are thus captured by it; or they spiral out of the unstable focus for  $R > R_H$  and thus escape from it. We refer to this type of behavior as neo-periodic capture and escape.

23. Mandel, P., Nonlinear Control in Optical Bistability, IEEE Quant. Opt., submitted.

We study the influence of a small periodic modulation of the input field amplitude in dispersive optical bistability. When the system is initially near one of the two limit points in a stable state, the addition of a small periodic modulation may either stabilize or destabilize the system. We prove that destabilization occurs as a result of critical slowing down when the modulation frequency is decreased.

24. Kriegsmann, G. A., An Asymptotic Theory of Rectification and Detection submitted.

The results of a systematic asymptotic analysis of the basic half wave rectifier and amplitude detector circuits are presented in this paper. A singular perturbation technique is applied directly to the governing differential equation and yields an asymptotic approximation of the output voltage as  $\omega RC \rightarrow \infty$ . Here,  $\omega$  is either the source frequency for the rectifier or the carrier frequency for the detector and  $RC$  is the time constant of the resistor-capacitor filter. The asymptotic result contains transient, ripple, and steady state information. The latter two components reduce to the standard

results when the diode's forward resistance is small, or equivalently, when the parameter  $\alpha = V_0 q / kT$  becomes large. Here,  $V_0$  is the peak value of the input voltage,  $q$  is the charge of an electron,  $k$  is the Boltzman constant, and  $T$  is the temperature. In particular, the ripple component of the rectifier output becomes a sawtooth wave in this limit. This is not an assumption to simplify the analysis but a consequence of the circuit's parameters as born out by the asymptotic analysis.

25. Kriegsmann, G. A. and Scandrett, C. L., Numerical Studies of Acoustic Pulse Scattering by Baffled Two-Dimensional Membranes, submitted.

A new method for solving the scattering problem of acoustic pulses by baffled membranes is described. It is an adaptation of the finite difference technique which uses an "artificial" boundary condition to mimic an infinite region. The numerical method is used on several problems involving a plane compact pulse. It is also used on a time harmonic pulse of infinite extent. This pulse generates a time-harmonic response as  $t \rightarrow \infty$ ; this is proved in Appendix A. The numerical scheme is marched out in time until the transients have decayed away and a time harmonic solution has been established. This "relaxation" scheme yields accurate solutions in a reasonable amount of time without requiring heavy or light fluid loading assumptions.

26. Ahluwalia, D. S., Kriegsmann, G. A. and Reiss, E. L., Scattering of Low Frequency Acoustic Waves Can Lead to Period Multiplication, J.A.S.A., in press.

The method of matched asymptotic expansions is used to study the scattering of plane, monochromatic, acoustic waves from baffled flexible surfaces in the limit as  $L/\lambda \rightarrow 0$ . Here  $\lambda$  is the wavelength of the incident acoustic wave and  $L$  is a characteristic size of the flexible surface, such as its maximum diameter. The baffled surface is either a membrane or a thin plate. Uniform asymptotic expansions of the scattered fields are obtained for both surfaces as  $L/\lambda \rightarrow 0$ . Thus, for example, it is valid in the near and far fields of the flexible surface. The method is applied to obtain the fields scattered by rectangular membranes and plates. It is found that plates are more efficient scatterers than "similar" membranes if the plates are sufficiently thin.

27. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Acoustic Pulse Scattering By Baffled Membranes, submitted Journal Acoust. Soc. of Amer.

Asymptotic expansions as  $\epsilon \rightarrow 0$  that are uniformly valid in  $t$  are obtained for the membrane's motion and the scattered acoustic pressure field. The small parameter is the density ratio of the acoustic fluid and the membrane. For simplicity of presentation, only plane, compact incident pulses are considered. The scattered field depends on the pulse's structure. If it is a sufficiently narrow band width pulse, then it is essentially reflected as though the baffled plane is completely rigid. However, if the pulse spectrum is sufficiently broad so that it contains one or more of the in-vacuo natural frequencies of the membrane, an additional scattered field is produced. This scattered field insonifies distant observation points after the rigidly reflected pulse has arrived. It is the sum of slightly damped, and oscillating outgoing spherical waves that represent the "decayed ringing" of the membrane. Application is given to the baffled circular membrane which is insonified by a normally incident pulse. Graphs of the membrane's motion and the far field acoustic pressure are given. They demonstrate the importance of the incident pulse width on the qualitative features of the response.

28. Sinay, L. R. and Reiss, E. L., Secondary Transitions in Panel Flutter: A Simple Model, to be submitted.

A simple, two degree of freedom mechanical model of panel flutter is presented. A perturbation method is employed to determine the secondary bifurcation of flutter states from divergence states, and divergence states from flutter states. The latter suggests a new method for controlling flutter by allowing small amplitude flutter and then increasing the flow velocity until secondary bifurcation into a divergence state occurs.



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